

Exploring Ring Flexagons

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Back in 1988 Peter Hilton and Jean Pedersen wrote *Build Your Own Polyhedra*, a book that taught us to do just that. Among the hexaflexagons, rotating rings, and Platonic solids, the authors gave directions for folding flat, ring-shaped polygons, each from just a straight strip of paper.

What Hilton and Pedersen didn't realize is that some of these objects *flex*, revealing hidden layers of paper and change shape to boot. They are flexagons that have been hiding in plain sight. This paper will explore two ring-shaped hexagons made from 30-60-90-degree triangles and show how they flex into symmetrical pinwheel shapes and collapse into a solid hexagon that also flexes. Figures 1 and 2 show these what these flexagons look like when they are first folded up. Note that the negative space inside flexagon A is a hexagon and the one inside flexagon B is star shaped.

Figure 1: Flexagon A

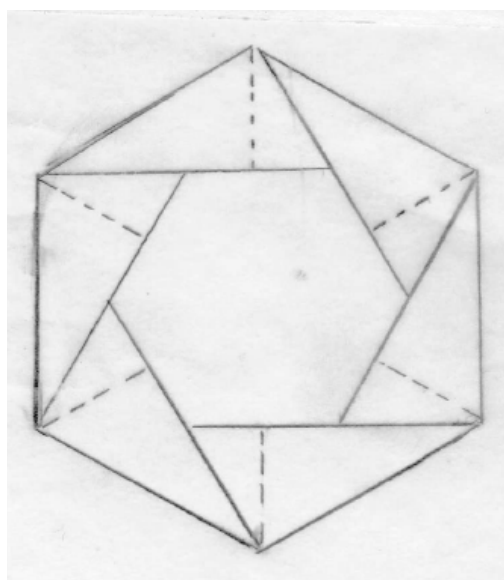
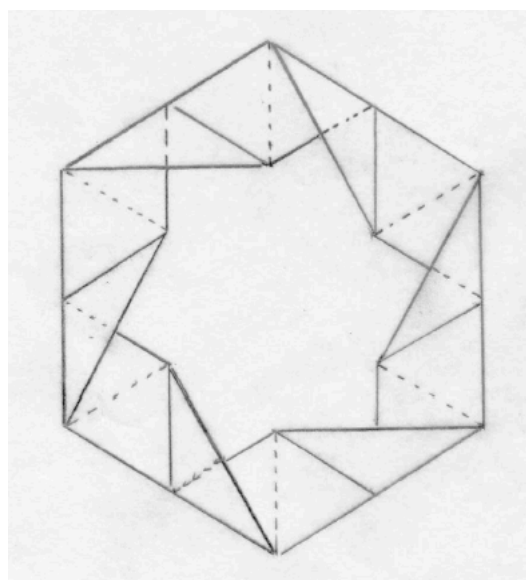


Figure 2: Flexagon B



This paper will briefly review some of the characteristics of the well-known hexaflexagon and then show how ring flexagons differ. For example, the faces of a hexaflexagon consist of triangles that virtually always stay together; that is, if one face has only blue triangles and another has only red, after flexing, you will not see a face containing both blue and red triangles. However, after flexing ring hexagon A or B, the resulting figure will have triangles of many colors, always in a symmetrical pattern. But how do we know

how to color the triangles on the straight strip that forms the ring flexagon? This paper will answer those questions.

Next we find that performing a flex—one very similar to the pinch flex used with the hexaflexagon—produces pinwheel shapes in both ring hexagons.

Figure 3: Pinwheel produced by Flexagon A

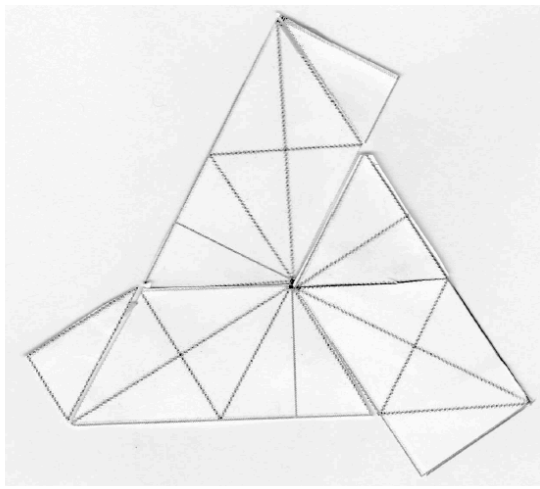
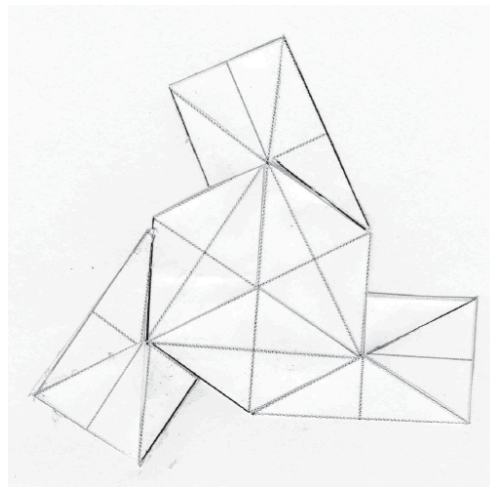


Figure 4: Pinwheel produced by Flexagon B



These ring flexagons do additional fascinating things. Flexagon B has another pinwheel shape up its sleeve and can also flex into a ring in which the center space is hexagonal. Both these ring flexagons can also flex into dodecaflexagons (solid hexagons divided into twelve 30-60-90 triangles). These dodecaflexagons can flex into small pinwheel triangles made of nine triangles, and into equilateral triangles made of eighteen 30-60-90 triangles.

Flexagons A and B also exhibit side exclusivity. Hexaflexagons—along with tetraflexagons composed of squares—have faces that are seen on both sides. But these ring-shaped hexagons must be flipped over to see every face.

Along with bringing these intriguing objects into the literature, this paper opens the door into a new area of flexagon inquiry: the ring flexagon. The easier-to-flex Flexagon A can be used to pique the mathematical curiosity of schoolchildren; the relationship between both flexagons and Möbius strips will be of interest to topologists. Many papers have explored the mathematical underpinnings of the hexaflexagon. For those mathematicians captivated by origami, the mysteries of these ring flexagons await.